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Probabilistic service life design of prefabricated concrete floor beams in Hungary

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RESEARCH OBJECTIVES

The importance of durability has recently become an important aspect of the design process. Main objective of the research was to provide a calculation method for the durability-design of pre-cast concrete members, that is:

- based on state of the art probabilistic approach with arbitrary number of stochastic parameters,
- considering the loads as a function of time,
- considering the decrease of load carrying capacity (F_u) in time due to slow deformations, degradation of structural geometry and aging of materials,
- fast and accurate enough for practical application,
- results are easy to use for practicing designers.

The research was focusing on the analysis of pre-fabricated, pre-stressed concrete girders because:

- they are widely used for the construction of residential houses and industrial buildings,
- the stochastic characteristics of process parameters (mean values and standard deviations of structural geometry and material properties) can be obtained from the results of quality control,
- manufacturing conditions of the members can be more precisely controlled and modified if necessary.


Airport maintenance base, Budapest


Prefabricated floor beams for residential buildings


Shopping centre, Nyíregyháza

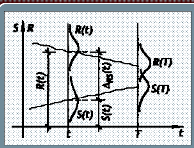

Large-span prefabricated beams for industrial buildings

PROBABILISTIC APPROACH FOR THE SERVICE LIFE DESIGN

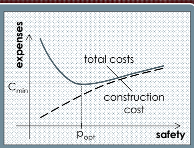
Durability: The structure is durable enough (functions properly) if the desired level (p_{opt}) for the probability of failure is not exceeded during its service life.

Probability of failure: The probability that the load effect (**S**) is exceeding the structural resistance (**R**).

p_G = prob[(R-S)<0]



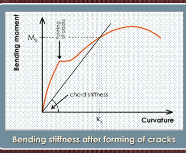
Changes of the stochastic distribution of external loads (S) and the structural resistance (R) in time causing the increase of failure probability.



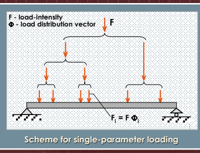
The interpretation of p_{opt} optimal safety. Total costs can be expressed as $C=C_c+C_f+D \cdot p_G$, where C_c is the construction cost, C_f is the maintenance cost and D is the property damage cost due to the failure with p_G probability.

Mean value of structural resistance ($F_{u,m}$) was calculated by Finite Element Method (FEM).

- Mean values of structural dimensions and material properties were used for the calculation
- Non-linear material model was used to describe the post-cracking behaviour of concrete
- The applied load was single-parameter load.
- The value of the load-intensity (F) was increased in steps until structural failure (crushing of concrete or splitting of steel bars) occurred.



Bending stiffness after forming of cracks



Scheme for single-parameter loading

Standard deviation of structural resistance (s_{Fu}, v_{Fu}) was calculated by Stochastic Finite Element Method (SFEM).

$$\delta K = \frac{\partial K}{\partial x} s_x$$ where s_x is the standard deviation of x.

The standard deviation of structural resistance was obtained as square root of the diagonal elements in the covariance matrix C_q:

$$C_q = \delta\tilde{q}\delta\tilde{q}^T = \tilde{K}^{-1} \frac{\partial K}{\partial \alpha} u \delta x C_r \delta x^T u^T \frac{\partial K^T}{\partial \alpha} \tilde{K}^{-T}$$

The correlation between different finite elements was described by an exponentially decaying function of the distance between the elements and the length of correlation.

$\tilde{q}$ – load vector including load intensity (F)
u – nodal displacements
K – stiffness matrix
C_r – correlation matrix
$\tilde{K}$ – stiffness matrix including F
C_q – covariance of load vector
δx – matrix including standard deviations of random input variables

VERIFICATION OF THE CALCULATION METHOD

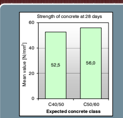
Products of 7 different Hungarian companies were considered during the determination of material properties to be used for the analyses:

Concrete strength:
- Compression tests on 150x150x150 mm cubes
- Altogether 732 specimens tested at age of 28 days
- 5 different concrete classes

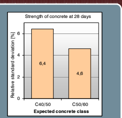
Steel bar strength:
- Tensile tests on altogether 291 specimens
- 3 different classes, 9 different diameters

Pre-stressing tendon strength:
- Tensile tests on 20 specimens
- 2 different types


Concrete cube specimen after compression test



Mean value of measured concrete strength



Standard deviation of measured concrete strength

Bending tests on 26 pre-stressed floor beams (type "EE")

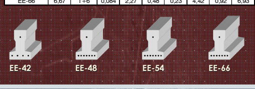
Bending tests were carried out on pre-stressed concrete floor beams to verify the numerical analysis on the stochastic characteristics of ultimate load (UL). Altogether 26 beams of type "EE" in 4 different sizes (effective length L between 4.20 and 6.60 m) were tested. Length of beam, height of the cross-section, width of the top and bottom flange, the concrete cover of the pre-stressing wires, as well as the ultimate load were measured in case of each beam.

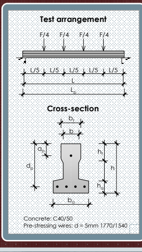
Average of measured values:

Type of beam	L [m]	Number of wires	L_{bm} [m]	h_m [mm]	$b_{f,m}$ [mm]	$b_{a,m}$ [mm]	$a_{p,m}$ [mm]	$d_{p,m}$ [mm]	$F_{u,m}$ [kN]
EE-42	4,27	1+4	4,40	189,8	80,8	144,1	37,8	168,5	49,28
EE-48	4,87	1+6	5,01	195,2	80,6	145,4	30,8	175,4	53,93
EE-54	5,47	1+6	5,64	196,1	81,7	144,4	39,5	176,5	51,60
EE-66	6,67	1+6	6,85	197,1	79,8	142,3	45,7	175,3	47,50

Relative standard deviation of measured values:

Type of beam	L [m]	Number of wires	v_{Lb} [%]	v_h [%]	v_{bf} [%]	v_{ba} [%]	v_{ap} [%]	v_{dp} [%]	v_{Fu} [%]
EE-42	4,27	1+4	0,171	1,60	3,13	1,59	18,21	1,73	6,92
EE-48	4,87	1+6	0,224	1,86	1,30	1,49	19,48	0,86	3,86
EE-54	5,47	1+6	0,119	2,72	1,96	1,98	11,32	2,07	6,32
EE-66	6,67	1+6	0,084	2,27	0,48	0,23	4,42	0,92	6,93

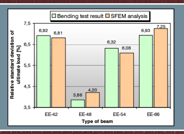




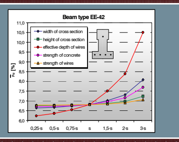
Verification of the calculation method

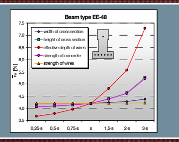
Comparison of the relative standard deviation of ultimate load (v_{Fu}) derived from bending tests and from numerical analysis:

Type of beam	v_{Fu} Bending test result [%]	v_{Fu} SFEM analysis [%]	v_{Fu} Difference between test and calculation [%]
EE-42	6,92	6,81	-1,6
EE-48	3,86	4,20	8,8
EE-54	6,32	6,08	-3,8
EE-66	6,93	7,25	4,6



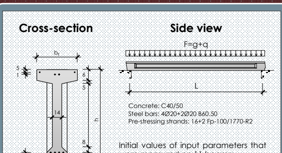
The effect of the standard deviation of different input parameters on the standard deviation of ultimate load was numerically analysed:





SERVICE LIFE DESIGN OF PREFABRICATED CONCRETE BEAMS

Several long-span, prefabricated concrete beams were analysed by the introduced probabilistic design method. Results of the analysis are demonstrated in case of the pre-stressed beam type "4000".



Initial values of input parameters that were measured on 11 beams:

	Number of strands	L	h_m	$b_{f,m}$
Mean value[mm]	2+16	27691	1442	604
Standard deviation [%]		0,028	0,533	1,177

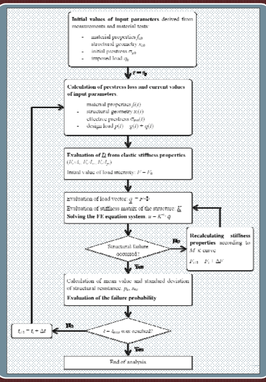

Reinforcement of beam "4000"

Effects that were considered during analysis:

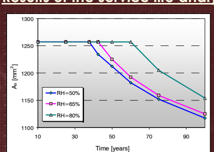
- Loss of initial pre-stress σ_{p0}
- Increase of standard deviation of geometrical sizes
- Decrease of mean value of material strength (concrete, steel bars, pre-stressing tendons)
- Increase of standard deviation of material strength
- Carbonation of concrete
- Carbonation induced corrosion of steel bars and tendons
- Change of mean value and standard deviation of loads

Varying conditions during the analysis:
- Time: t = 10, 25, 50, 75 and 100 years
- Relative ambient humidity: RH = 50%, 65% and 80%
- Initial value of imposed load (at t=0): q_0 = 16, 18, 20, 22 and 24 kN/m

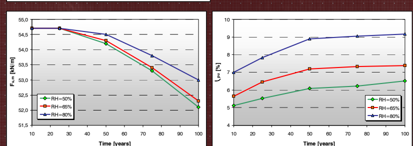
Flowchart of the calculation process:



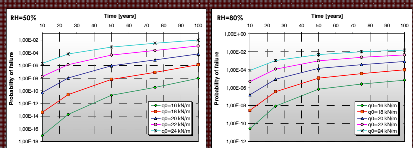
Results of the service life analysis



The effect of the ambient relative humidity on the process of carbonation and corrosion. In case of lower humidity levels, the process of carbonation is faster so the corrosion of steel bars starts earlier. Higher levels of humidity result in slower carbonation thus corrosion starts later. However, higher level of humidity also means higher rate of corrosion after the depassivation.



Change of the mean value and relative standard deviation of structural resistance in time at different humidity levels.



Change of the probability of failure (p_G) in case of different initial q_0 imposed loads and RH relative ambient humidity levels.

The above charts can be used for the service life estimation of beam "4000" under given circumstances!